Project 1:

Part 1: Aitken’s delta-squared process

Problem 1: Consider the problem of approximating the root of using Fixed-Point iteration.

Part A: Show that the root of is also a fixed point of the function .

Begin with the function . Finding the root is equivalent to finding the value of which satisfies the equation:

(1)

Rewriting equation (1) gives:

(2)

(3)

Defining gives:

(4)

Finding the value of which satisfies is equivalent to finding the fixed point of . Because this is the same value of that gives , the root of is the same as the fixed point of .

Part B: Use the Fixed-Point Theorem to show that Fixed-Point Iteration with will converge to for any in the interval .

Theorem 2.4 states that if (i.e., for all ), exists on , and there exists some constant such that , then for any the sequence converges to a unique fixed point .

Because is a decreasing monotone function (i.e., ), if and , then .

(1)

(2)

(3)

(4)

Thus, both and are in the interval .

Consider and its derivative . Both of these functions are defined for all , and are continuous functions for real number inputs. Notably, this means that exists on the interval because . Because is a monotone increasing function, will have the largest value on the interval at . This gives:

(1)

Thus, satisfies the condition

(2)

because

(3)

Therefore, because has been shown to satisfy all the conditions of Theorem 2.4 on the interval , where and , will converge to a unique fixed point for any .

Part C: Write a MatLab script to implement Fixed-Point Iteration with and . Use as the stopping criterion.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 1 | 0.67032 | 0.27032 |
| 2 | 0.51154 | 0.15878 |
| 3 | 0.59957 | 0.088024 |
| 4 | 0.54905 | 0.05052 |
| 5 | 0.5775 | 0.028451 |
| 6 | 0.5613 | 0.016199 |
| 7 | 0.57047 | 0.0091664 |
| 8 | 0.56526 | 0.0052052 |
| 9 | 0.56821 | 0.00295 |
| 10 | 0.56654 | 0.0016737 |
| 11 | 0.56749 | 0.00094903 |
| 12 | 0.56695 | 0.00053831 |
| 13 | 0.56725 | 0.00030528 |
| 14 | 0.56708 | 0.00017314 |
| 15 | 0.56718 | 9.8194e-05 |
| 16 | 0.56712 | 5.5691e-05 |
| 17 | 0.56715 | 3.1584e-05 |
| 18 | 0.56714 | 1.7913e-05 |
| 19 | 0.56715 | 1.0159e-05 |
| 20 | 0.56714 | 5.7617e-06 |
| 21 | 0.56714 | 3.2677e-06 |
| 22 | 0.56714 | 1.8533e-06 |
| 23 | 0.56714 | 1.0511e-06 |
| 24 | 0.56714 | 5.9611e-07 |

Code can be found in Appendix A.

Part D: Show that the solution obtained using Fixed-Point Iteration converges linearly to by showing that

(0)

for the 4 largest values of .

For the iteration above, use .

For :

(1)

For :

(2)

For

(3)

Note that this quotient equals zero for because the value of is taken to be the value of , which was the first that satisfied the error requirement. It is difficult to determine from these points alone whether holds. Calculating this quotient for all gives a more accurate picture. When accounting for all , it is clearer that the quotient is approximately constant.

|  |  |
| --- | --- |
|  |  |
| 1 | 0.53886 |
| 2 | 0.58321 |
| 3 | 0.55804 |
| 4 | 0.57232 |
| 5 | 0.56418 |
| 6 | 0.56886 |
| 7 | 0.5661 |
| 8 | 0.56786 |
| 9 | 0.56652 |
| 10 | 0.56787 |
| 11 | 0.56606 |
| 12 | 0.56894 |
| 13 | 0.56406 |
| 14 | 0.57258 |
| 15 | 0.55768 |
| 16 | 0.58411 |
| 17 | 0.53811 |
| 18 | 0.62109 |
| 19 | 0.48028 |
| 20 | 0.74799 |
| 21 | 0.32537 |
| 22 | 1.3102 |
| 23 | 0 |

Code can be found in Appendix A.